## Calculations Document

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

Strategies for calculation need to be supported by familiar models and images to reinforce understanding. When teaching a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the concept.

The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.
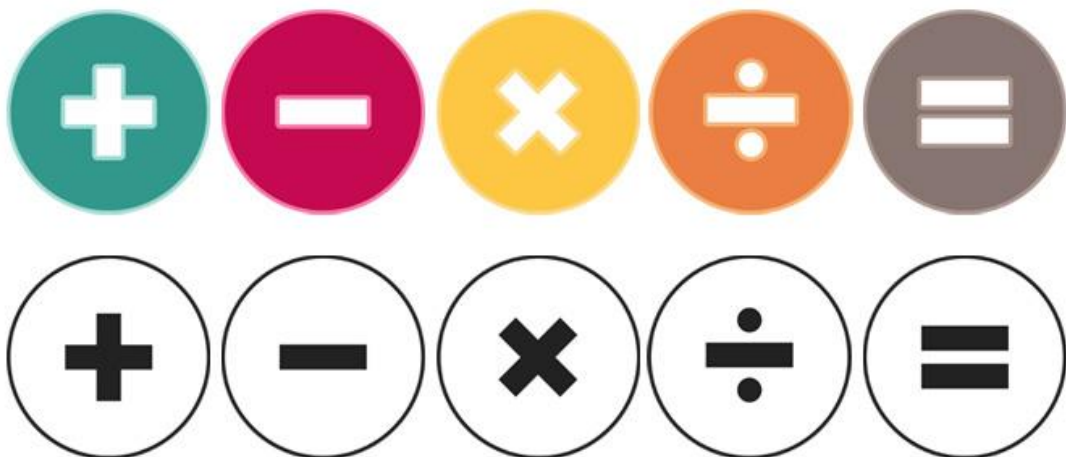
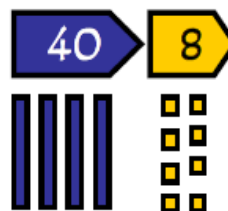# Progression in Teaching Addition and Subtraction

Mental Skills
Recognise the size and position of numbers
Count on and back in ones and tens
Know number bonds to 10 and 20
Know number facts for all numbers to 20
Add and subtract multiples of 10 to any number
Partition and recombine numbers
(only partition the number to be subtracted)
Bridge through 10

Models and Images
Counting apparatus
Place value apparatus
Place value cards
Number tracks
Numbered number lines
Marked but unnumbered number lines
Empty number lines
Hundred square
Counting stick
Bead string
Models and Images charts
ITPs – Number Facts, Ordering Numbers, Number Grid
Counting on and back in ones and tens

Key Vocabulary: Addition	Key Vocabulary: Subtraction
add	subtract
addition	take away
plus	minus
and	count on, count back
count on	less
more	fewer
sum	difference between
total	decrease
altogether	
increase	

Begin to count forwards and
backwards in
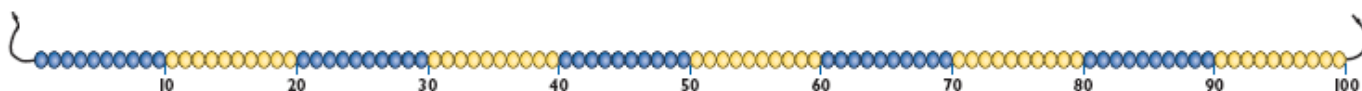familiar contexts such as
number rhymes or stories

Continue to count forwards and
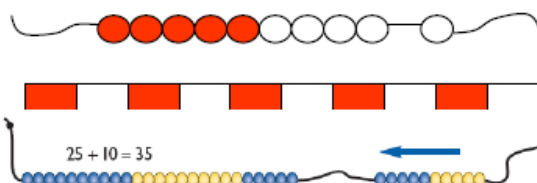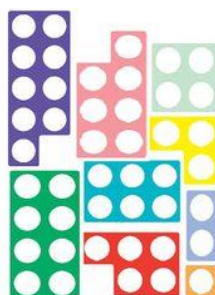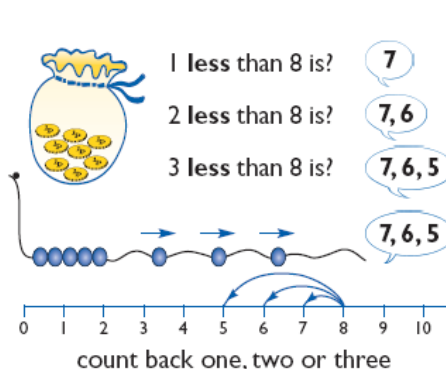back in ones from any given number
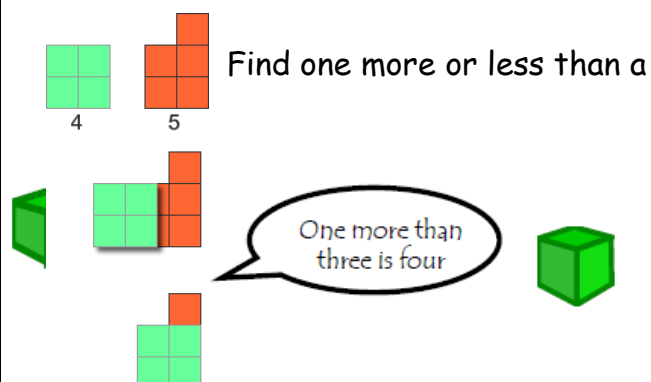
Count forwards and back in tens

Add and subtract
using concrete
objects

$25 + 10 = 35$

Find one more or less than a   number

4   5

One more than
three is four

I **less** than 8 is?   7
2 **less** than 8 is?   7, 6
3 **less** than 8 is?   7, 6, 5

7, 6, 5
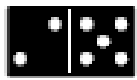
count back one, two or three

Begin to relate addition to
combining two groups of
objects

and   makes 5
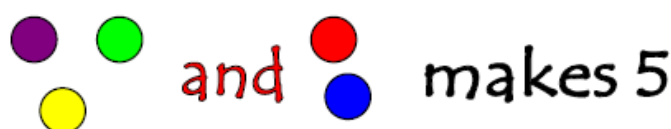
Begin to use the + and = signs to record
mental calculations in a number sentence

$3+2=5$

Know that addition can be
done in any order
(Commutative law)

$2 + 5 = 7$ 2 count on 5

$5 + 2 = 7$ 5 count on 2
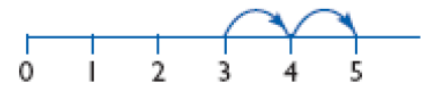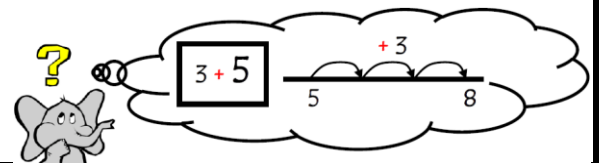
1 + 2 = 3

2 + 1 = 3

Count along a number line to
add numbers together

Put the biggest number first and count on

? $3 + 5$ + 3

Begin to relate subtraction to ' taking away '

Begin to use the – and = signs to record
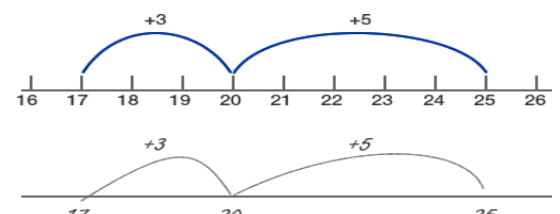mental calculations in
a number sentence

5-2=3

Count backwards along a number line to
'take away'

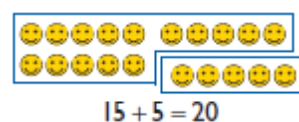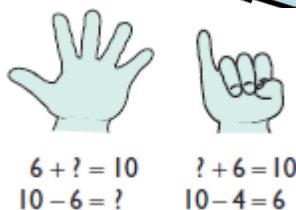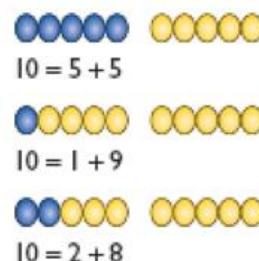Count up to find the difference

The difference
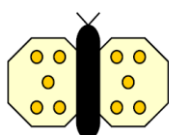between 11
and 14 is 3.
$14 – 11 = 3$
$11 + \square = 14$

+3   +5

16 17 18 19 20 21 22 23 24 25 26

+3   +5

Know by heart all pairs of numbers with a total of 10 and 20

$6 + ? = 10$ $? + 6 = 10$
$10 - 6 = ?$ $10 - 4 = 6$

$15 + 5 = 20$

Know by heart subtraction facts for numbers up to 10 and 20

6 and how many more make 10?
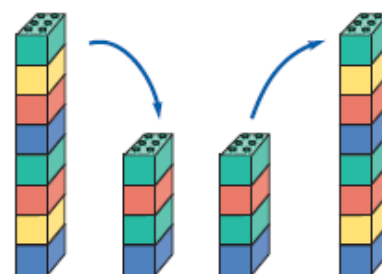$6 + \square = 10$

$10 = 5 + 5$

$10 = 1 + 9$

$10 = 2 + 8$

Know doubles and halves of numbers (Inverse)

$5 + 5 = 10$

$1 + 1 = 2$
double 1 is 2

$2 - 1 = 1$
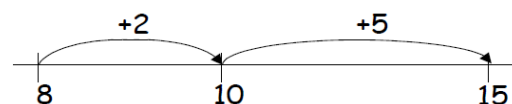half of 2 is 1

half of 8 is 4
$8 \div 2 = 4$

double 4 is 8
$4 \times 2 = 8$

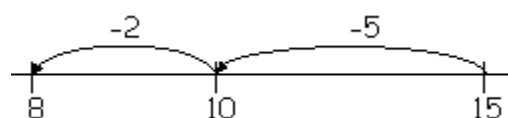Add two single-digit numbers that bridge 10

$8 + 7 = 15$

+2   +5
8   10   15

Subtract single digit numbers often bridging through 10

$15 - 7 = 8$

-2   -5
8   10   15

Begin to partition numbers in order to add

53
50 3

10p 10p 10p $30p + 4p = 34p$
1p 1p 1p 1p
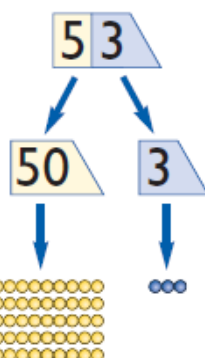
Begin to partition numbers in order to take away

Adding two two-digit numbers
Partitioning and recombining
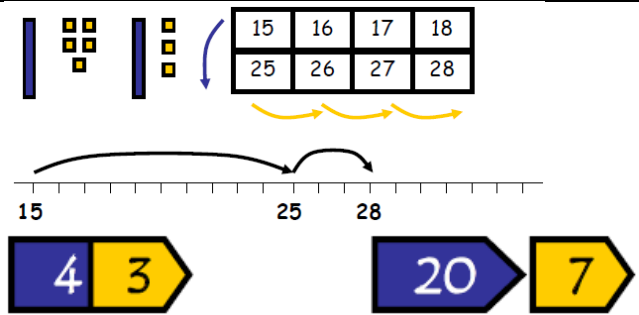
Partitioning number to be subtracted
with exchanging
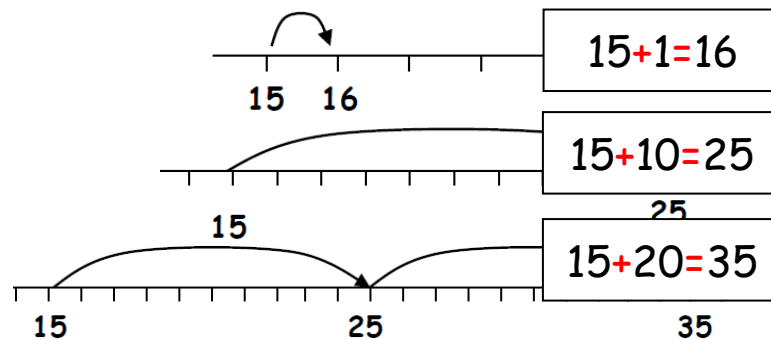23 – 7 = 1 6
(links to counting back on number line)

43-27=16

43 – 20 = 2 3
23 – 7 = 1 6
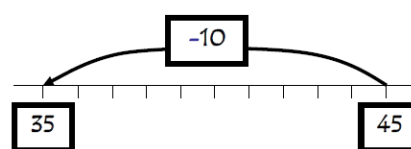
Add 10 to a
two-digit number

15+1=16

15+10=25

15+20=35

Subtract 10 from a
two-digit number

45-10=35

Know which digit changes when adding or subtracting 1s or 10s to and from any number

Add and subtract multiples of
10 to and from any number

25 + 10 = 35
35… 45, 55

35, 45, 55…   35 + 20 is 55

67 – 10 = 57

67, 57, 47, 37…

67 – 30 is 37   67…57, 47, 37

23p
33p, 43p…

23p and 10p more is 33p
and 10p more makes 43p

54p in the purse. Take 10p
out, another 10p and so on

54p
44p, 34p…
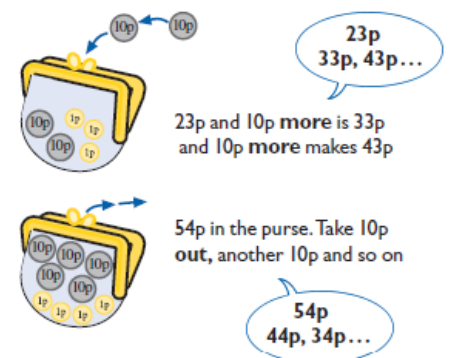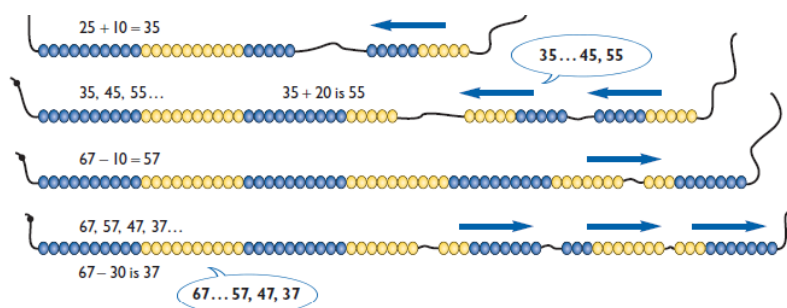
Expanded method
It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method.

48 + 36

T   U

40 + 8
30 + 6
_____
80 + 4
10

48
+ 36
____

43 - 27 = 16

³⁰4̶0̶ + ¹⁰⁺3
− 20 + 7
_____
   10 + 6

to subtract 7 units
we need to exchange
a ten for ten units
·············▶
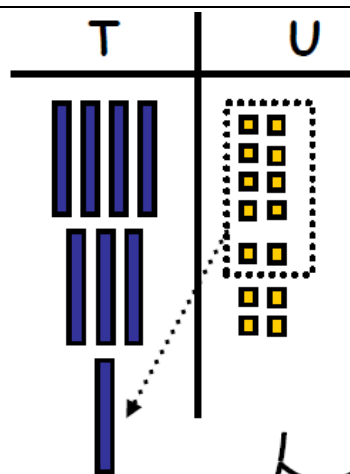
Standard written method

The previous stages reinforce what happens to numbers when they are adding and subtracted using more formal written methods. It is important that the children have a good understanding of place value and partitioning.

```
  4 8
+ 3 6
_____
  8 4
  1
```

```
 ³4̶ ¹3
-  2 7
_____
   1 6
```

From the Primary National Curriculum, Appendix 1 (edited)

Addition and subtraction

789 + 642 becomes	874 – 523 becomes	932 – 457 becomes

7 8 9
+ 6 4 2
_____
1 4 3 1
   1 1
	8 7 4
	− 5 2 3
	_____
	   3 5 1
		8 12 1
		   9̶ 3̶ 2
		− 4 5 7
		_____
		   4 7 5

Answer: 1431	Answer: 351	Answer: 475

Progression in Teaching Multiplication and Division

Mental Skills

Recognise the size and position of numbers

Multiplication and division facts for 2, 3, 5, 10

Count from 0 in multiples of 4, 8, 50, 100

Count from 0 in multiples of 6,7,9, 25 and 1000
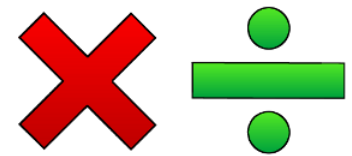
Double and halve numbers up to 10, 20 then beyond

Recognise multiplication as repeated addition

Recognise division as repeated subtraction

Quick recall of multiplication and related division facts

Use known facts to derive associated facts

Multiply and divide by 10, 100, 1000 and understanding the effect

Models and Images

Counting apparatus

Place value apparatus

Arrays

100 squares

Number tracks

Numbered number lines

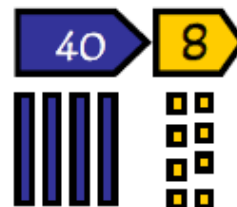Marked but unnumbered lines

Empty number lines.

Multiplication grids

Counting stick

Bead strings

Models and Images charts

ITPs – Multiplication grid, Number Dials, Multiplication Facts

Key Vocabulary: Multiplication	Key Vocabulary: Division
lots of	lots of
groups of	groups of
times	share
multiply	group
multiplication	halve
multiple	half
product	divide
once, twice, three times	division
array, row, column	divided by
double	remainder
repeated addition	factor
	quotient
	divisible

Count in

2s, 5s, 10s

from zero (groups of)		
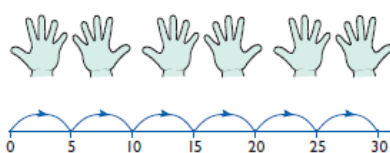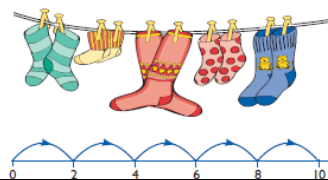
Understand multiplication as repeated addition

Begin to use the x and = signs to record mental calculations in a number sentence
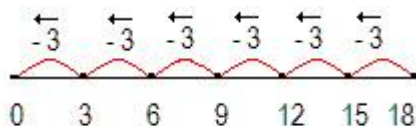
$2 + 2 + 2 + 2$

$2 + 2 + 2 + 2 = 8$
$4 \times 2 = 8$
2 multiplied by 4
4 lots of 2

18

15 — -3

12 — -3

9 — -3

6 — -3

3 — -3

0 — -3

Understand division as repeated subtraction
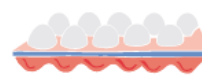
Begin to use the ÷ and = signs to record mental calculations in a number sentence

-3 -3 -3 -3 -3 -3
0   3   6   9   12 15 18

Understand multiplication as an array

$4 \times 2 = 8$
$2 \times 4 = 8$

$2 \times 4 = 8$
$4 \times 2 = 8$

$5 \times 2 = 10$
$2 \times 5 = 10$

$2 \times 5 = 10$
$5 \times 2 = 10$

Understand division as sharing

Understand division as grouping

(Use arrays as shown)

$3 \times 2 = 6$

$6 \div 3 = 2$

Multiplication and division on a number line.

(How many 3s in 15? 15 ÷ 3 = 5   show commutativity)

2 hops of 5

5 hops in 15. How big is each hop?

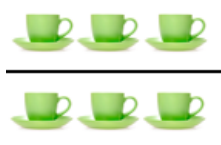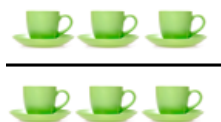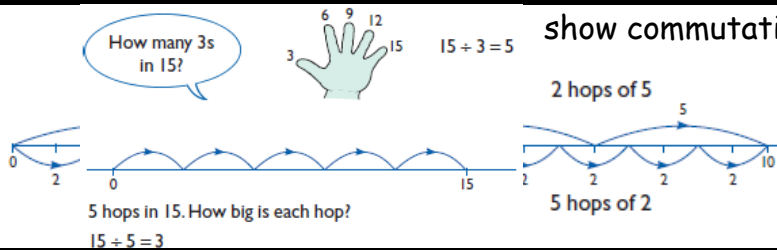15 ÷ 5 = 3

5 hops of 2

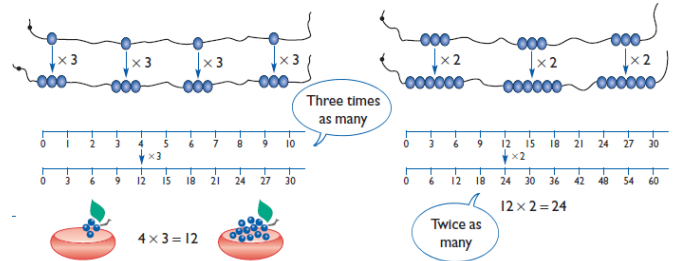
Multiplication and division facts for 2, 3, 5, 10

Count from 0 in multiples of 4, 8, 50, 100

Count from 0 in multiples of 6, 7, 9, 25 and 1000

Recall mutiplication and division facts for multiplication tables up to 12X12

Three times as many

$4 \times 3 = 12$

Twice as many

$12 \times 2 = 24$

Use known facts to work out new ones

Twice as many

$12 \times 2 = 24$
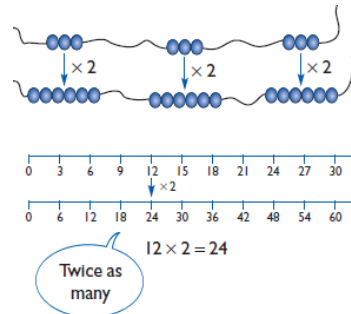
Understand that …

24 x 20 = 24 x 2 x 10

24 x 50 = 24 x 5 x 10

Use place value apparatus to support multiplication of TU x U

10   3

4
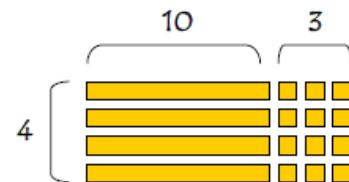
Use place value apparatus to support the multiplication of TU x U alongside the grid method

13 x4 =___

10   3

4   40   12

10   3

4   40   12

40 + 12 = 52

23 x4 =___

10   10   3

4

10   10   3

4   40   40   12

20 (2 x 10)   3

4   80   12

80 + 12 = 92
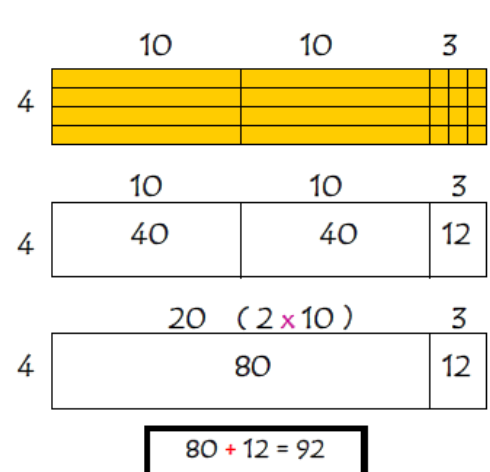
Multiplying TU x TU

33 **x** 14 **=**

	30	3	
10	300	30	= 330 **+**
4	120	12	= 132

462

Standard written method
From the Primary National Curriculum, Appendix 1

Short multiplication

24×6 becomes

```
    2  4
×      6
  1  4  4
     2
```
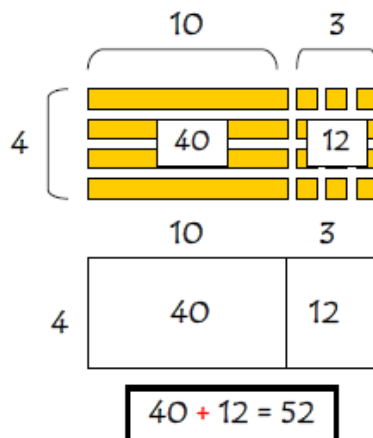
Answer: 144

342×7 becomes

```
    3  4  2
×         7
  2  3  9  4
     2  1
```

Answer: 2394

2741×6 becomes

```
  2  7  4  1
×           6
1  6  4  4  6
   4  2
```

Answer: 16 446

Long multiplication

24×16 becomes

```
      2
    2  4
×   1  6
  2  4  0
  1  4  4
  3  8  4
```

Answer: 384

124×26 becomes

```
    1  2
  1  2  4
×     2  6
2  4  8  0
   7  4  4
3  2  2  4
1  1
```

Answer: 3224

124×26 becomes

```
    1  2
  1  2  4
×     2  6
   7  4  4
2  4  8  0
3  2  2  4
1  1
```

Answer: 3224

Standard written method

From the Primary National Curriculum, Appendix 1

Short division

98 ÷ 7 becomes

```
      1  4
  7 | 9  8
      ²
```

Answer: 14

432 ÷ 5 becomes

```
      8  6  r 2
  5 | 4  3  2
         ³
```

Answer: 86 remainder 2

496 ÷ 11 becomes

```
        4  5  r 1
  1 1 | 4  9  6
           ⁵
```

Answer: $45\frac{1}{11}$

Long division

432 ÷ 15 becomes

```
          2  8  r 12
  1  5 |  4  3  2
         3  0  0
         ─────────
         1  3  2
         1  2  0
         ─────────
            1  2
```

Answer: 28 remainder 12

432 ÷ 15 becomes

```
          2  8
  1  5 |  4  3  2
         3  0  0    15×20
         ─────────
         1  3  2
         1  2  0    15×8
         ─────────
            1  2
```

$\frac{12}{15} = \frac{4}{5}$

Answer: $28\frac{4}{5}$

432 ÷ 15 becomes

```
          2  8 · 8
  1  5 |  4  3  2 · 0
         3  0 ↓
         ─────────
         1  3  2
         1  2  0 ↓
         ─────────
            1  2  0
            1  2  0
         ─────────
                  0
```

Answer: 28·8